Biogeodynamics and Earth System Sciences Summer School (BESS)

Data Assimilation for the Lorenz (1963) Model using Ensemble and Extended Kalman Filter

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OUTLINE

Data Assimilation

Kalman Filter (EnKF and EKF)

Lorenz Model

Results of Sensitivity Tests

Future Challenges

DATA ASSIMILATION

Data Assimilation is usually defined as

"Estimation and prediction (analysis) of an unknown true state by combining observations and system dynamics (model output)"

It is needed in order to:

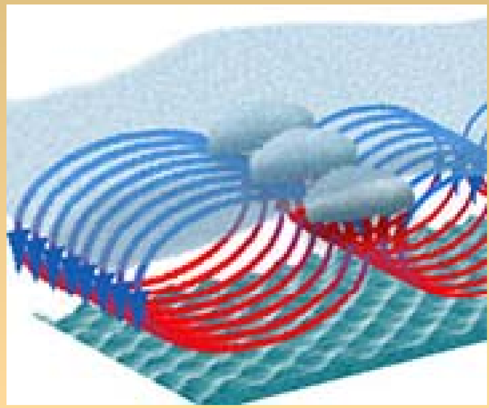
- Reduce uncertainties and biases
- Improve forecasting
- Estimate initial state of a system (e.g. hydrologic system) from multiple sources of information
- Permit forecast adjustments

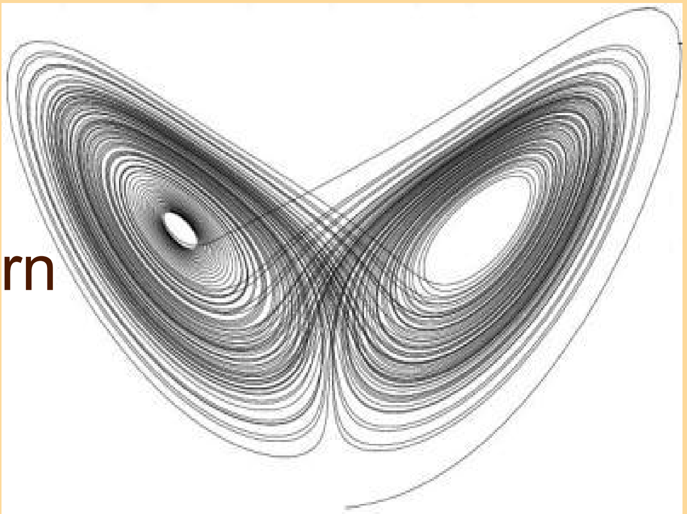
EXAMPLE

Lorenz's Model

a simplified model of thermal convection

in the atmosphere.





pattern

Outcome:

- No predictable

- Butterfly effect

A BIT OF MATHS...

- The Lorenz model
- Bistability and chaotic behaviour

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = x(\rho - z) - y$$
$$\frac{dz}{dt} = xy - \beta z$$

- Matlab code to simulate the model dynamics
- Perturbation of a ”true run” with a random noise to get ”pseudo-observations”

Where:

For the bistable behaviour:

$\beta = 8/3$, $\rho = 1.01$, $\sigma = 10$

For the Lorenz attractor:

$\beta = 8/3$, $\rho = 28$, $\sigma = 10$

A BIT MORE MATHS

Kalman Filter

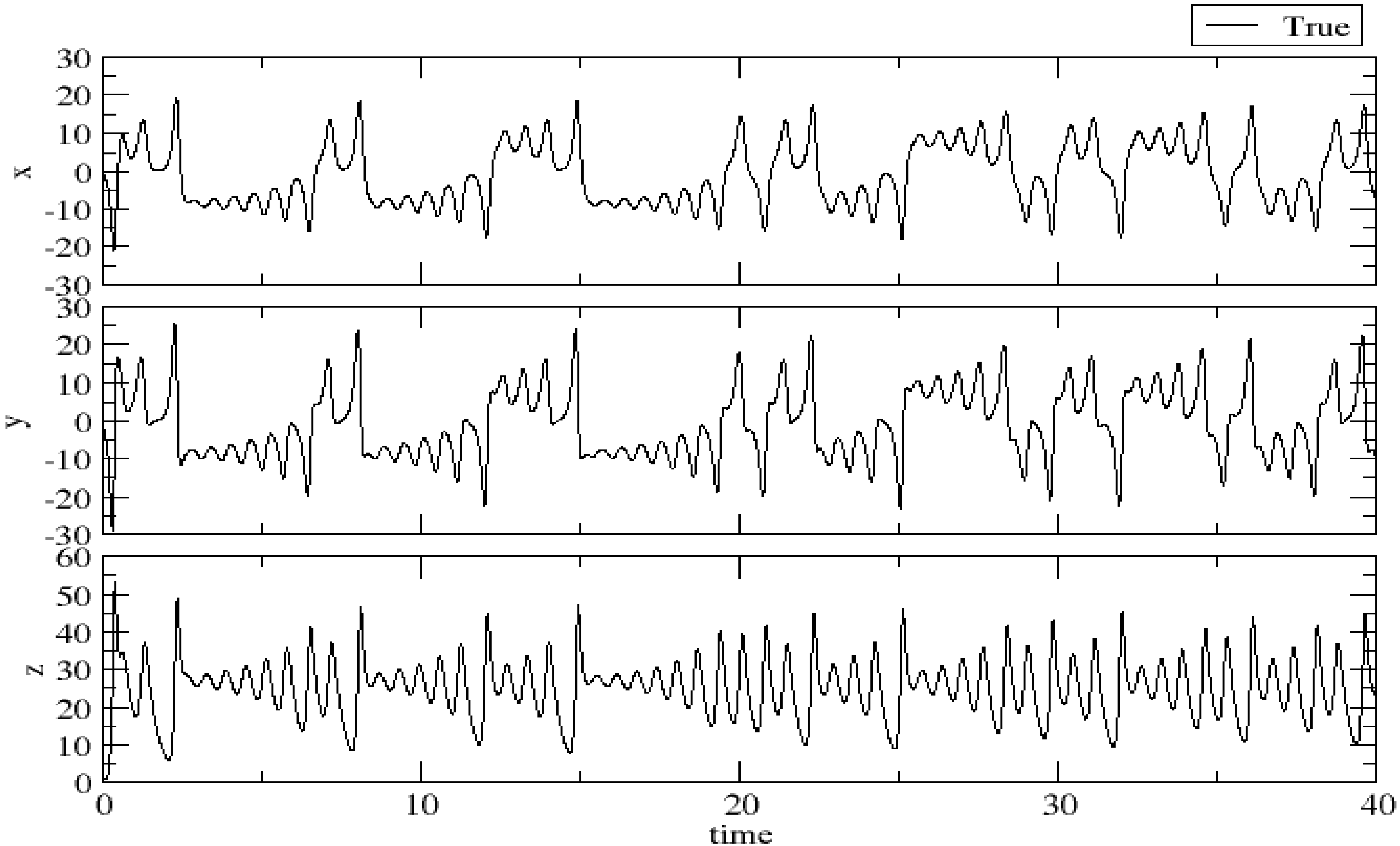
Predict

Predicted (*a priori*) state estimate	$\hat{\mathbf{x}}_{k\|k-1} = \mathbf{F}_k \hat{\mathbf{x}}_{k-1\|k-1} + \mathbf{B}_k \mathbf{u}_k$
Predicted (*a priori*) estimate covariance	$\mathbf{P}_{k\|k-1} = \mathbf{F}_k \mathbf{P}_{k-1\|k-1} \mathbf{F}_k^{\mathrm{T}} + \mathbf{Q}_k$

Update

Innovation or measurement residual	$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k\|k-1}$
Innovation (or residual) covariance	$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k\|k-1} \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k$
Optimal Kalman gain	$\mathbf{K}_k = \mathbf{P}_{k\|k-1} \mathbf{H}_k^{\mathrm{T}} \mathbf{S}_k^{-1}$
Updated (*a posteriori*) state estimate	$\hat{\mathbf{x}}_{k\|k} = \hat{\mathbf{x}}_{k\|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k$
Updated (*a posteriori*) estimate covariance	$\mathbf{P}_{k\|k} = (I - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k\|k-1}$

RESULTS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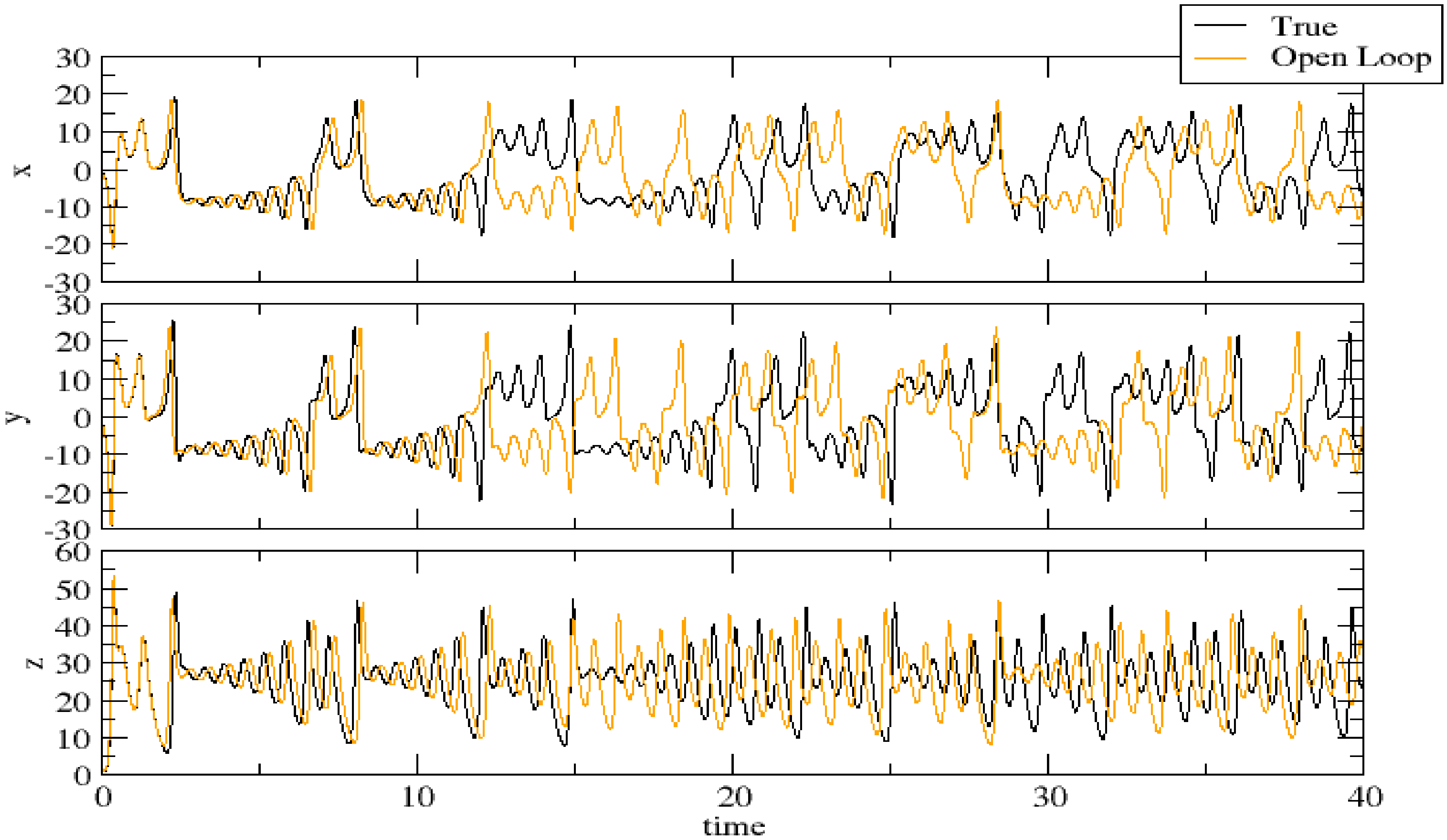
time

RESULTS



True
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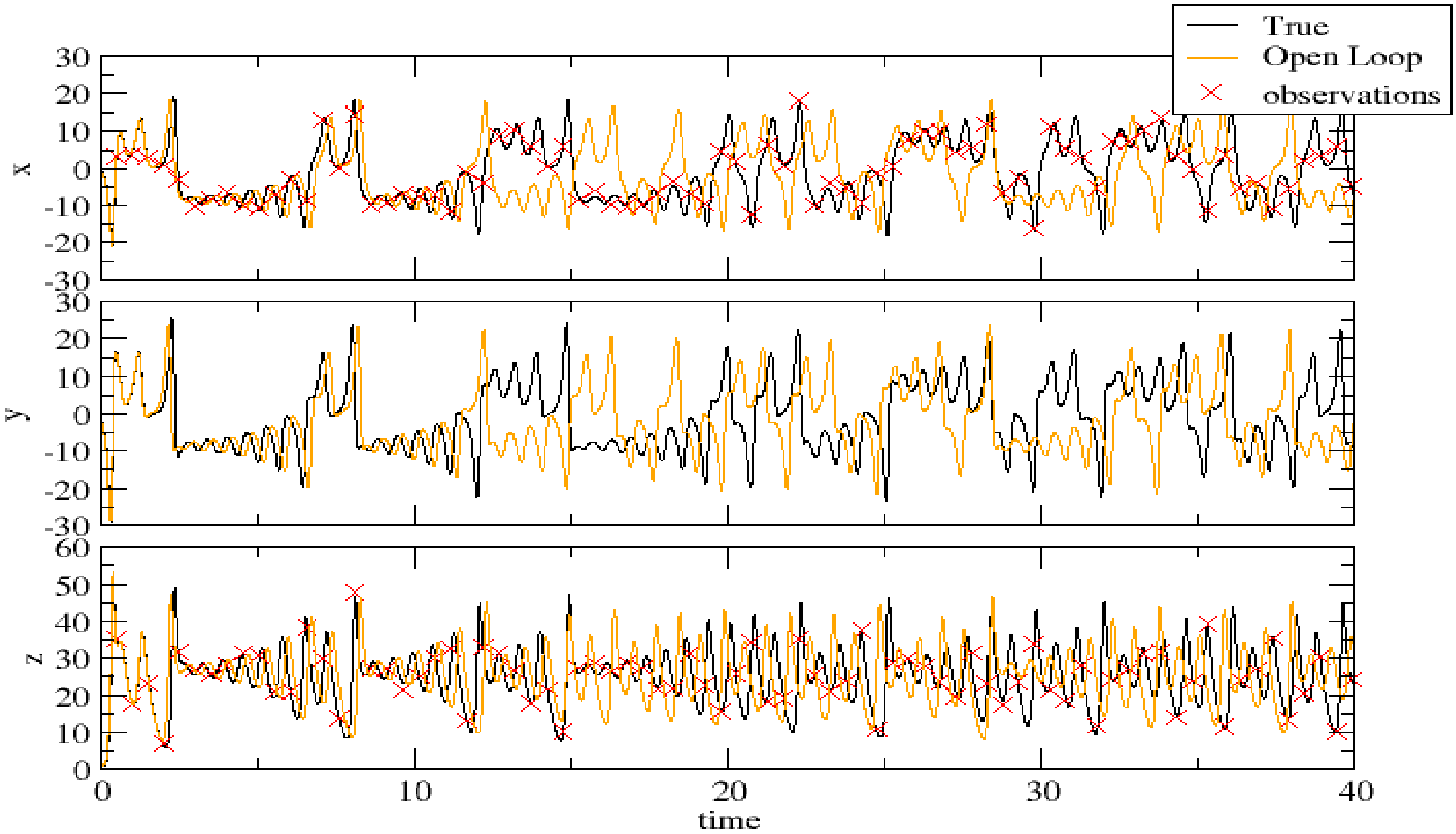
time

RESULTS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observations
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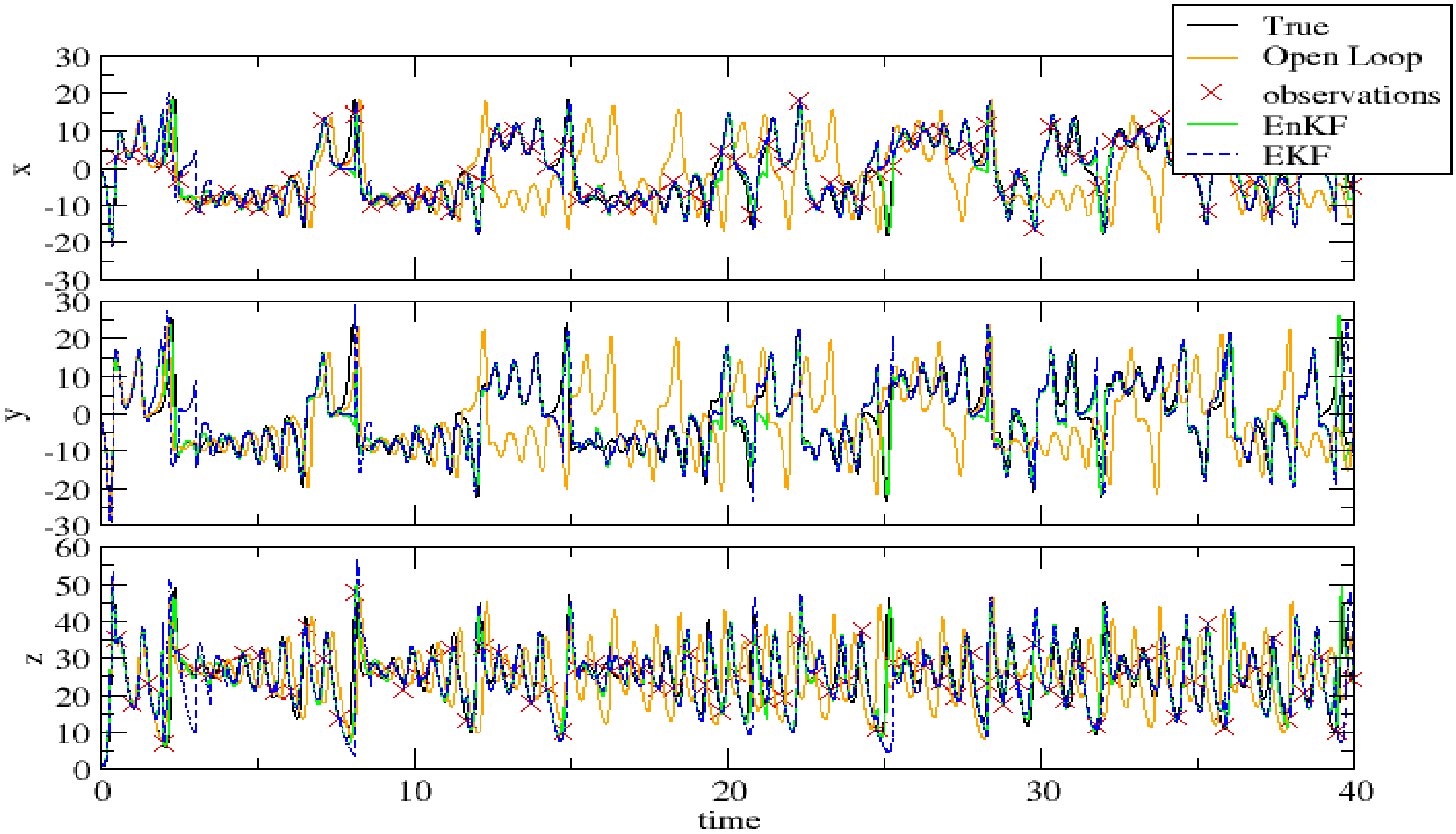
RESULTS



True
Open Loop
observations
EnKF
EKF
x
y
z
time
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-20
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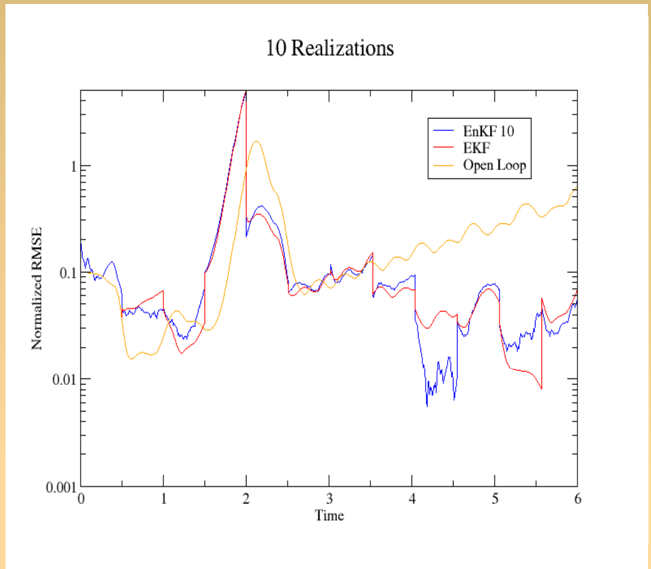
SENSITIVITY TESTS:
Number of realization of the EnKF

10



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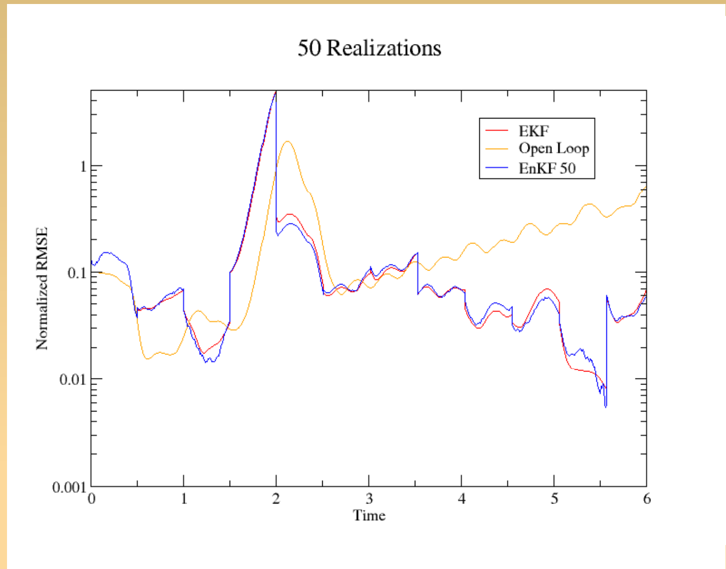

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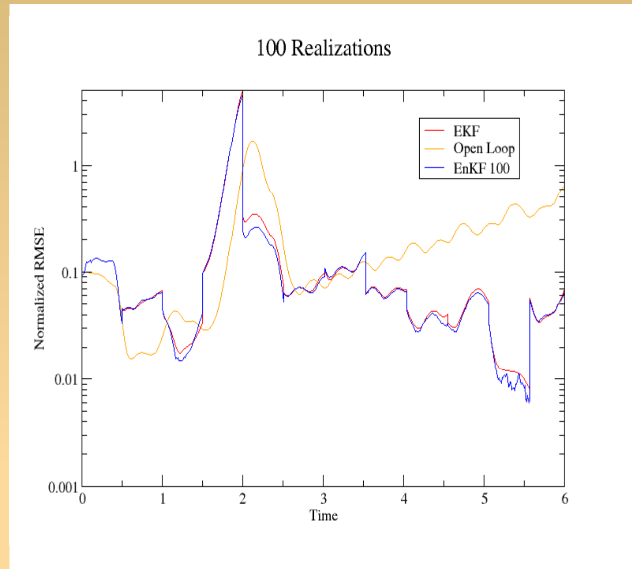
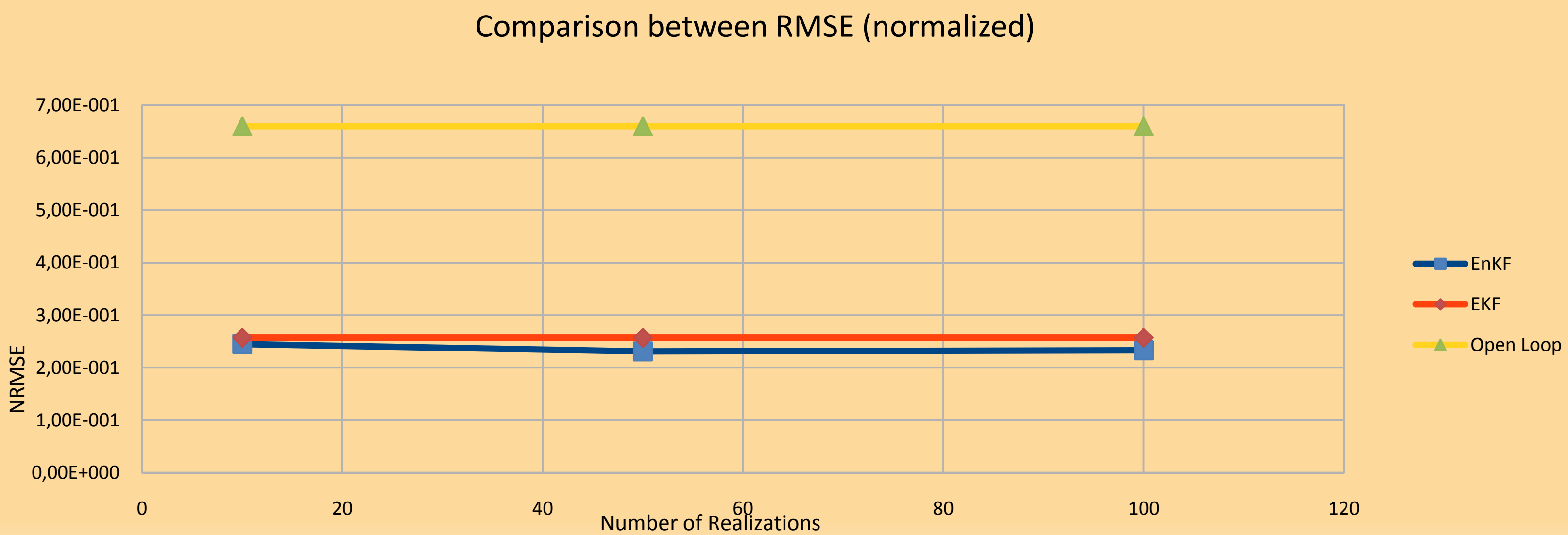
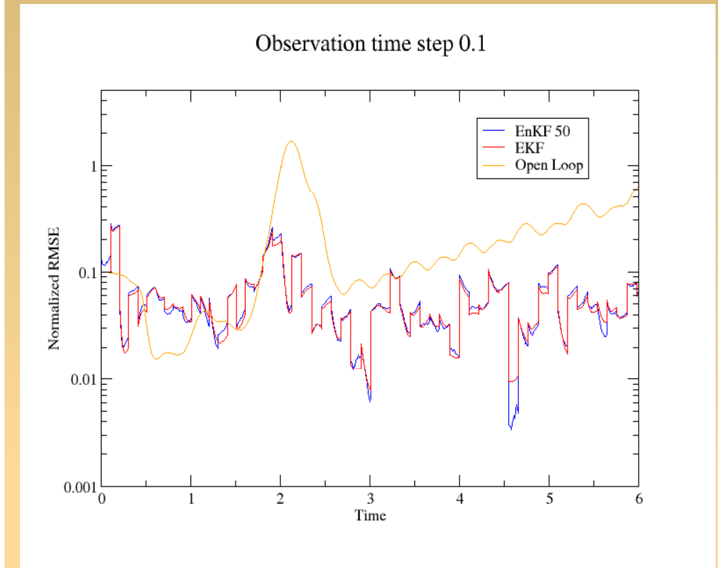


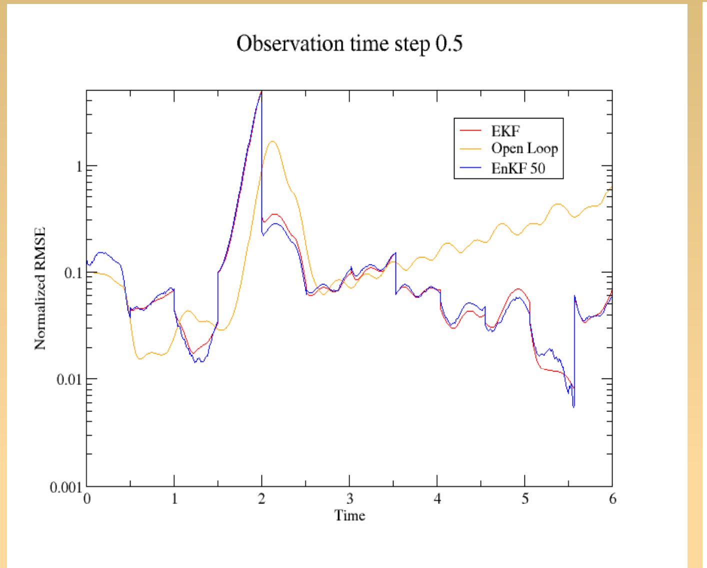


SENSITIVITY TESTS: Observation Time Step

0.1



0.5



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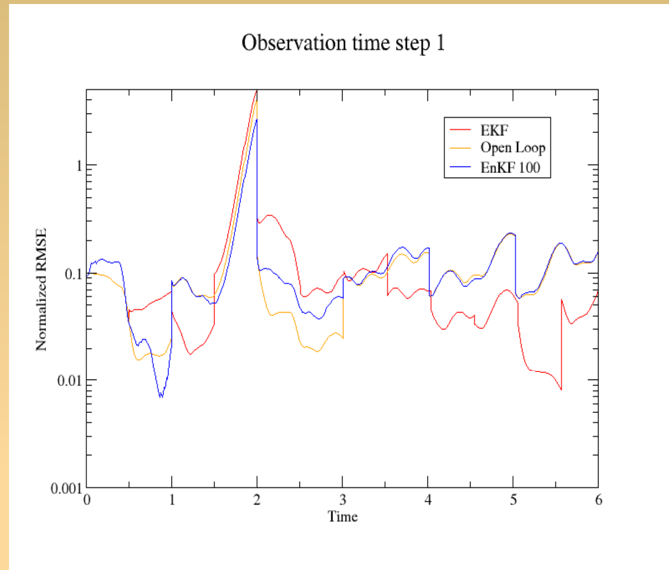

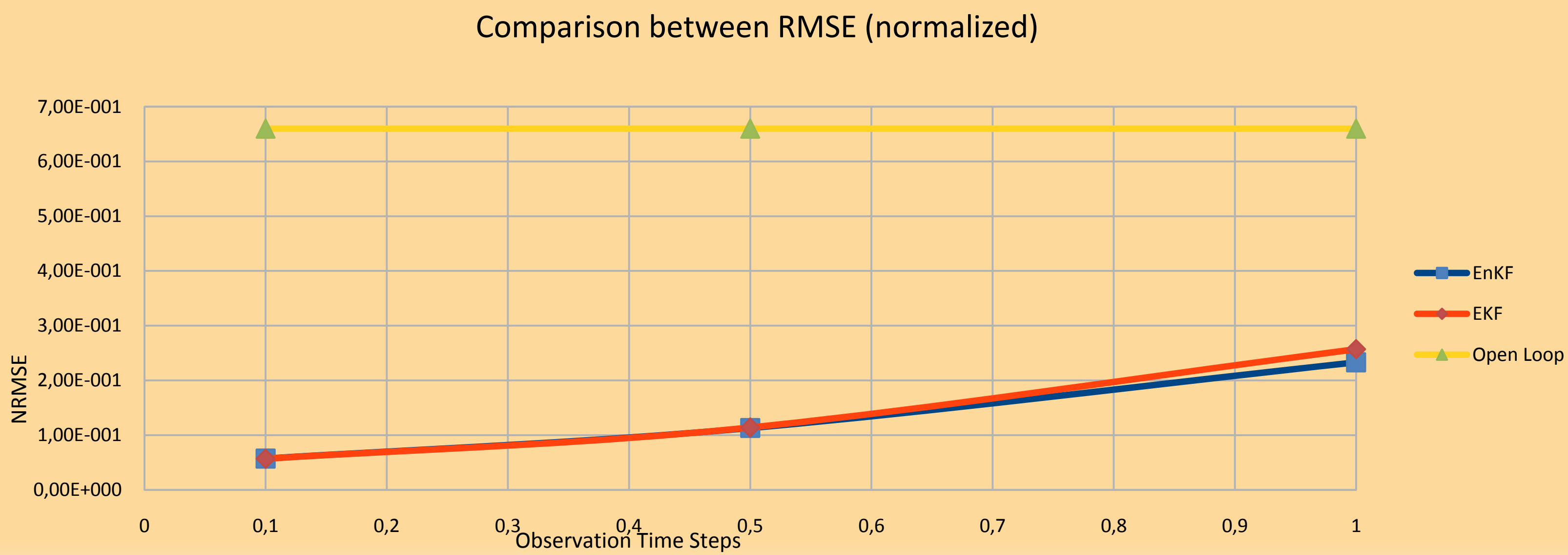

CONCLUSIONS

Data Assimilation on the Lorenz Model using EnKF and EKF was performed:

- Sensitivity tests on the number of realization on the EnKF show that N=10 is already optimal for this small model
- Sensitivity tests on the observation time steps show the error increases as the amount of information provided decreases
- EnKF and EKF perform similarly but EKF is to be prefered because computationally more efficient

FUTURE CHALLENGES

- Further sensitivity tests
- (e.g. ”bistable dynamics”)
- Parameter estimation
- Utilize Data Assimilation for ”real problems” (e.g. Weather predictions)

Data Assimilation for the Lorenz model using Ensemble and Extended Kalman Filter

Thanks for your attention